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LETTER TO THE EDITOR

Non-Schrödinger forces and pilot waves in quantum cosmology†

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Abstract. I argue that the version of the pilot wave interpretation of quantum mechanics which uses a non-local non-Schrödinger force is inconsistent when applied to distributions with small numbers of particles. Thus, no version of the pilot wave interpretation (sometimes called the de Broglie–Bohm, or causal, interpretation) can be applied to the wavefunction of quantum cosmology because in any version of this interpretation, there is only one particle, the universe.

There seems to be a disagreement among the adherents of the pilot wave/causal interpretation of quantum mechanics as to what the fundamental assumptions of the theory actually are. Bell considered the causal interpretation to be one of the class of '... theories in which a linear Schrödinger equation is held to be exactly and universally correct. There is then no "jumping", no "reducing", no "collapsing", of the wave function. Two such theories will be analyzed, one due to de Broglie (1960) and Bohm (1952) ...' (Bell 1981, p 613). In contrast, Dewdney *et al* (1986) lump together as versions of the 'Copenhagen interpretation' all theories which adhere to the completeness assumption, in which the wavefunction governed by Schrödinger's equation represents the most complete description possible of an individual system. Dewdney *et al* tell us that in the causal interpretation '... the particle is conceived of as an inhomogeneity in a Madelung fluid undergoing random fluctuations about the Madelung mean. The velocity will then not be exactly $\nabla S/m$ nor the density exactly $|\psi|^2$ but these values hold as averages' (Dewdney *et al* 1986, p 368). Bohm and Hiley (1984), in contradiction to Dewdney *et al*, assert: 'Clearly, the quantum potential interpretation is to be distinguished from a hydrodynamic model (such as that of Madelung). For, in this model, the particle is simply pushed mechanically by the fluid' (Bohm and Hiley 1984, p 260).

In a previous article (Tipler 1984), I pointed out that there were problems with the version defended by Bell, Bohm, Hiley and others; in particular, I showed that it could not apply to quantum cosmology. In the present letter, I shall show that similar problems occur in the version defended by Dewdney *et al*, the version which requires a force not present in the Schrödinger equation—that is, a force not included in the 'quantum potential' of the Bohm–Bell version. Together, the present letter and the

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previous article show that no version of the pilot wave interpretation can be applied to the cosmological case. I shall refer to the version of the pilot wave interpretation analysed here as the non-Schrödinger force/pilot wave (NS/PW) interpretation.

There is first of all a conceptual problem with the application to quantum cosmology of the NS/PW interpretation as it has been developed to date. If a non-Schrödinger force is essential to an interpretation, then this force will be the dominant factor at the Planck scale of 10^{-33} cm when quantum gravity is important. In particular, we would expect the fluctuations of a Madelung fluid to become more important the smaller the scale, and the Planck length is a lower bound for its fluctuation length scale. Since in the current NS/PW model there is no theory for the fluctuations of the non-Schrödinger force (Jammer 1974, p 36), if the said force existed we would know only that the equations of quantum gravity are probably meaningless in the very regime in which quantum gravitational effects would be significant. Thus the current NS/PW interpretation cannot be applied in the Planck epoch; it cannot be used to interpret the wavefunction governed by the Wheeler–DeWitt equation, since it claims that this equation is meaningless in the primary regime we would want to interpret the wavefunction of the universe.

I shall now review my argument against Bell's version of the pilot wave interpretation, and I shall show that a modification of the argument will rule out any possibility of applying the NS/PW version to quantum cosmology.

It is very interesting that Dewdney *et al* (1986), the main defenders of the NS/PW interpretation, agree with my assessment of Bell's version of the pilot wave interpretation: it will not work. We also agree about the reason why it will not work: the pilot wave interpretation cannot consistently assume that the particle distribution ϕ equals the quantum mechanical probability density $|\psi|^2$ at all times. If ϕ is measurably different from $|\psi|^2$ at some arbitrary initial instant, then this difference will be preserved by the time evolution if Schrödinger's equation holds exactly (and if the particle velocities are always given by $\nabla S/m$, with no crossing of trajectories) and the measured particle distribution will be different from the one predicted by quantum mechanics and actually observed. There is a nice illustration of this in the paper of Dewdney *et al* (1986) (see also Dewdney and Hiley 1982). In their example of scattering from a semi-transparent surface with $\psi_0 = \exp[-(x-0.5)^2/2\sigma_0^2]\exp(-ik_0x)$, if we choose $\phi_0 = |\psi_0|^2$ for $x > 0.5$ but $\phi_0 = 0$ for $x < 0.5$, then all the particles will be transmitted and none reflected; the inequality $\phi_0 \neq |\psi_0|^2$ will be preserved and measured by the total transmission. An even better example is illustrated in figure 3 of Philippidis *et al* (1979): it shows that, if ψ_0 is uniform in two slits and zero elsewhere, then the characteristic two-slit interference pattern is seen if the particle distribution ϕ_0 is also uniform in the sense that the distance between nearest neighbours is the same for all particles in a given slit. It also shows that, if we modify ϕ_0 from uniformity—say, by removing some of the particles pictured in figure 3 of Philippidis *et al* (1979)—while keeping ψ_0 the same, then the interference pattern can be made to disappear. The particle trajectories in all figures in the above-mentioned articles are obtained by assuming that the particle velocity is exactly $v = \nabla S/m$, and that at all times, the nth particle has some precise position coordinate $x_n(t)$.

These results illustrate the theorem I proved in Tipler (1984): if Schrödinger's equation and $v = \nabla S/m$ hold *exactly*, then $\phi_0 \neq |\psi_0|^2$ will result in a theory experimentally different from quantum mechanics, and measurably wrong. Thus Bell's version of the pilot wave interpretation will be refuted if we can show that the logic of his interpretation requires $\phi_0 \neq |\psi_0|^2$.

There are two reasons why Bell's version requires $\phi_0 \neq |\psi_0|^2$, both of which I discussed at length in Tipler (1984). These are summarised here.

First, if the particles really have precise positions, then necessarily the particle distribution is a sum of delta functions:

$$\phi(t) = \sum \delta[x_n(0) - x_n(t)] \tag{1}$$

where the summation is over all the particles in the system. There is simply no way this sum of delta functions is going to equal, either initially or later in time, the smooth function $|\psi|^2$. This will also be true if we imagine that the particles are not points, but finite-sized objects, so that the spatial distribution of matter is differentiable, and thus so is $\phi(x, t)$. For even in this case, the particle distribution will be represented by a collection of sharp spikes, so that ϕ_0 will vary rapidly in the vicinity of a particle, while $|\psi_0|^2$ will vary much more slowly there. This phenomenon is nicely illustrated by Dewdney *et al* (1986) in their example of scattering from a semi-transparent surface. There are 21 particles initially incident on their semi-transparent surface, so at any time there are 21 spikes in the actual particle distribution. These spikes have finite thickness—represented by the thickness of the lines—and the spikes are closer together near $x = 0.5$ than at $x = 0.55$ or $x = 0.45$, the closeness near $x = 0.5$ representing the attempt of Dewdney *et al* (1986) to match $|\psi_0|^2$, which is a Gaussian distribution. But in fact the Gaussian distribution is not matched by the actual particle distribution; in fact, we have ϕ_0 quite different from $|\psi_0|^2$, so in reality the model of Dewdney *et al* is not self-consistent. Also, if the particles really have precise positions, there is no way Dewdney *et al* can make their model self-consistent and in agreement with the predictions of quantum mechanics. Dewdney *et al* (1986) themselves admit the truth of this, but in effect claim it applies only to the Bell version of the pilot wave interpretation, and not to the NS/PW version. However, this admitted fact does invalidate the Bohm–Vigier 'demonstration' that $\phi(x, t) \to |\psi|^2$ with time: Bohm and Vigier (1954) tacitly assumed that the spikes in the ϕ distribution will smear out with time. This is impossible whatever version of the pilot wave interpretation one uses, for it is a consequence only of the assumption that '... the individual particles do have precise trajectories in spacetime...' (Dewdney *et al* 1986, p 366), which Dewdney *et al* (and I) regard as the hallmark of this interpretation. Particles may jump streamlines in the Dewdney *et al* version of the pilot wave interpretation, but they do *not* smear out. Since the Bohm–Vigier 'demonstration' that $\phi(x, t) \to |\psi|^2$ rapidly with time is crucial to the validity of the NS/PW version—indeed, it is crucial to any version of the pilot wave interpretation, for if $\phi(x, t) \neq |\psi|^2$ on a sufficiently large scale for a sufficiently long time, the predictions of the pilot wave interpretation will be inconsistent with observations—this means the NS/PW version is also invalid.

But there is a second reason why ϕ cannot equal $|\psi|^2$ in the NS/PW interpretation: any real system must consist of only a finite number of particles. Dewdney *et al* tell us '...the essentially new features of quantum mechanics revealed in the causal interpretation is that the motion of individual particles depends not only on their initial positions but also on the quantum state ψ of the whole system.... The new features are demonstrated in the form of the quantum potential which determines the particles' motion. The form of this potential depends in the single particle case on the quantum state ψ of the whole system and so quantum motions are not determined simply by specifying the initial positions. ... [The initial wavefunction] ψ_0 characterizes the outcome of preparation process (this could be a system such as a source, collimator, shutter, slit)' (Dewdney *et al* 1986, p 369). A quantum field which acts non-locally

on the particles of the system, and which is determined by the structure of the entire system, may in fact exist. However, an acceptable physical theory must tell us exactly how this field depends on the system as a whole, and it is easy to see that the pilot wave interpretations fail to do so in a consistent way. What do the above quoted statements about the nature of ψ_0 really mean mathematically? We can see what they really mean mathematically by looking at exactly how Dewdney *et al* constructed their scattering example. First, an initial wavefunction ψ_0 is chosen and then particles are placed at $t=0$ in a spatial distribution that closely matches the distribution $|\psi_0|^2$. The system, then, consists of a certain number of particles, together with the semi-transparent surface. The non-local nature of the quantum field is made manifest if we imagine the scattering example with the pictured 21 particles being generated by 21 successive scattering experiments, each with just one particle. The trajectory of each particle is determined not only by its initial position and momentum, but also by the initial positions and momenta of the other 20 particles, and by structure of the semi-transparent surface. In effect, each particle 'knows' about the past and future scattering of all the other particles, even though some particles have not yet even been placed in the experimental apparatus. This is certainly non-classical non-local behaviour, but such precognitive (Bell 1986) particle motion may be allowed in principle. However, any real experiment will not reproduce the initial Gaussian distribution of the scattering example exactly, because a real experiment will have only a finite number of particles: in the Dewdney *et al* example, there are only 21. In the cosmological case, there is only one 'particle' represented by a wavefunction, for there is only one universe. After a finite number of particles are inserted into an experimental apparatus, the apparatus is disassembled. At this point, the system has ceased to exist and future particles can no longer influence the motion of particles in the past. Therefore, the actual wavefunction representing the total system must have originally been the wavefunction corresponding to the actual finite distribution of particles that actually went through the apparatus during the time it existed. In the Dewdney *et al* scattering example this is the actual sample of 21 particles. By representing the wavefunction as a Gaussian and not the actual approximate Gaussian, Dewdney *et al* are in effect assuming that the 'system' consists of an uncountable number of particles which would be required to generate a true smooth Gaussian. Thus, not only are the particles interacting non-locally with the 21 real particles, they are also interacting with an infinite number of imaginary particles! Surely, this is idealistic philosophy with a vengeance! The only way to avoid idealism is to regard the quantum field ψ as a real field which is not completely determined by all the particles in the system, and in particular for which $\phi \neq |\psi|^2$. This inequality is particularly blatant in the cosmological case, since at all times the density of particles $\phi(x, t)$ consists of only a single trajectory in superspace (or minisuperspace (Misner 1974)), that of a single unique universe. As we saw above, the inequality $\phi \neq |\psi|^2$ leads to a disagreement with the predictions of quantum mechanics.

But Dewdney *et al* (1986) tell us that in the NS/PW version we would expect this inequality to hold for only a short time, for the non-Schrödinger fluctuating force causes the particles to cross the flow lines determined from $v = \nabla S/m$, and this force makes a ϕ quite different from $|\psi|^2$ to approach quickly the latter on average. However, in all their computer-generated examples (Dewdney *et al* 1986, Bohm and Hiley 1984, Dewdney and Hiley 1982, Philippidis *et al* 1979), the defenders of the NS/PW interpretation always assume that $v = \nabla S/m$ generates the actual trajectories; they never insert the fluctuating force. They claim that this fluctuating force (which they never insert)

is so strong that in the scattering example, if in fact '... all initial positions were in fact in the rear half of the packet, say, nevertheless the predicted probabilities will be $\phi=|\psi|^2$ given by quantum mechanics...' (Dewdney *et al* 1986, p 369).

But if there were only a *single* trajectory—as there would be in cosmology, since there is only one universe—no fluctuating force could smear it out. The fluctuating force might cause the trajectory of the universe to deviate from the trajectory given by $v=\nabla S/m$, but it cannot, by the basic assumption of *all* versions of the pilot wave interpretation, multiply trajectories. The non-Schrödinger force may cause the universe's trajectory to bounce around from the rear half to the front half of the wave packet, but this single trajectory will always be in one half or the other. In the cosmological case, we must always have ϕ very unequal to $|\psi|^2$. The single unique trajectory in cosmology shows the Bohm–Vigier 'demonstration' that $\phi(x,t)\rightarrow|\psi|^2$ with time, even if it were valid for microscopic systems, cannot apply in cosmology, since no non-Schrödinger force can give $\phi(x,t)\rightarrow|\psi|^2$ if $\phi(x,t)$ consists of only a single trajectory.

Although I do not have a proof for the non-cosmological case, I do not think Dewdney *et al* can explicitly exhibit a non-Schrödinger force that would have the properties they claim. Certainly, they never attempt to exhibit such a force; all trajectories they compute are obtained from $v=\nabla S/m$, and they never even try to do a self-consistent calculation with ϕ_0 in the rear half of their assumed wave packet, and *show* that, in fact, their assumed non-Schrödinger force indeed forces ϕ to approach $|\psi|^2$. My feeling is that, if the fluctuating force were as strong as Dewdney *et al* assume, it would be sufficiently strong to erase the interference pattern in the double slit experiment analysed in Philippidis *et al* (1979). I would expect that only if the force has the property that it is essentially zero for ϕ near $|\psi|^2$ can it possibly erase the non-quantum distribution while preserving the quantum interference, but only if there are a near infinity of particles can ϕ be near $|\psi|^2$, as I showed above. The distribution ϕ cannot be near $|\psi|^2$ in any real experiment. In particular, I personally think that Dewdney *et al* could not do a self-consistent computation including the non-Schrödinger force using a mere 21 particles, the number of particles in their calculation without the non-Schrödinger force. However, I do not claim to have demonstrated this inconsistency in this non-cosmological case—my demonstration applies only to cosmology where there is only one trajectory—so I invite defenders of the NS/PW interpretation to attempt a self-consistent calculation including the non-Schrödinger force.

I think defenders of the NS/PW interpretation misunderstand the reason why wave packet collapse is considered necessary in the standard interpretations of quantum mechanics. For example, Dewdney *et al* (1986) refer to the work of Cini (1983) as having shown that wave packet collapse is a redundant hypothesis in the formalism of quantum mechanics. Cini himself points out that his arguments are essentially those of Gottfried (1966); I myself think the Gottfried arguments are basically those of Daneri *et al* (1962) (see also Loinger 1968, Jammer 1974). Cini and the others (and hence Dewdney *et al*) claim that the measurement problem is solved if it can be shown that the wavefunction is split into wave packets which essentially do not interfere. This claim has been refuted by Jauch *et al* (1967) and Bub (1968). The basic idea of the refutation is simply stated. Let us model a measurement of a system which can have only two states (i.e. wave packets), $|\uparrow\rangle$ and $|\downarrow\rangle$, say. Let the measuring apparatus be in its neutral state $|n\rangle$ initially, and let the measuring apparatus go from $|n\rangle$ to $|u\rangle$ if the system is in the eigenstate $|\uparrow\rangle$ and from $|n\rangle$ to $|d\rangle$ if the system is in the eigenstate

$|\downarrow\rangle$. If the system is not in an eigenstate, but in a linear superposition, then by the linearity of Schrödinger operators, the effect of the measurement operator M must be

$$M(a|\uparrow\rangle + b|\downarrow\rangle)|n\rangle = a|\uparrow\rangle|u\rangle + b|\downarrow\rangle|d\rangle. \tag{2}$$

That is, the measuring apparatus is itself in a linear superposition of states; the state of the universe is not either $|\uparrow\rangle|u\rangle$ or $|\downarrow\rangle|d\rangle$, but rather (2). Hence the system trajectory is not in either the wave packet $|\uparrow\rangle$ or the wave packet $|\downarrow\rangle$, but rather it is in both. If we accept the completeness of the quantum mechanical formalism, then no other conclusion is possible. Since, in the standard interpretations, the system trajectory *is* in one wave packet or the other and not in both, it follows that one packet or the other has in actuality collapsed and that the quantum mechanical formalism is in fact incomplete. To put it another way, in the standard interpretations and in the NS/PW interpretation, the system *must* be in only one of the non-overlapping wave packets and the formalism cannot tell us which. The measurement problem is not solved until we have some additional theory which does tell us which. It is this additional theory which the NS/PW seeks to provide, but cannot.

Fortunately, quantum cosmology does make sense without wave packet collapse. If we accept a realist view of quantum mechanics—that all elements of the formalism correspond to a real entity—we are led inescapably to the Everett many-worlds interpretation (MWI) of quantum mechanics, which is not a version of the Copenhagen interpretation, as for example Dewdney *et al* (1986) claim. Deutsch (1985a, b) in fact has shown how the MWI can be experimentally distinguished from the Copenhagen interpretation. The Copenhagen interpretation involves a distinction between classical and quantum levels, as Bohm and Hiley (1985) have pointed out, but the MWI considers *all* levels to be quantum levels. Thus even macroscopic observers can behave quantum mechanically—which cannot happen in the Copenhagen interpretation—and this feature can be used to obtain testable differences between the MWI and the Copenhagen interpretation. The largest level is the cosmological level, and only an interpretation, such as the MWI, that treats all levels equally can be used in quantum cosmology and can be used to interpret the wavefunction of the universe (Tipler 1986a, b, Vilenkin 1986). I have defended the MWI at length elsewhere (Tipler 1986a, b, Barrow and Tipler 1986) and pointed out in these articles how the MWI can be used to interpret the wavefunction of the universe.

Most advocates of the various versions of the pilot wave interpretation misunderstand the MWI. For example, both Bell (1981) and Bohm and Hiley (1984) tell us that the MWI involves 'the assumption of a nondenumerable infinity of universes, with a corresponding nondenumerable infinity of observers within them' (Bohm and Hiley 1984). Actually, the number of worlds is determined by the size of the interpretation basis (see Deutsch 1985a). The number is necessarily denumerable (if quantum mechanics is defined on a separable Hilbert space) and almost always finite (see Tipler 1986a, b, Barrow and Tipler 1986). Another major misconception is that the entire universe splits whenever something is observed. This is, of course, silly; as Einstein put it in a meeting (Wheeler 1979) with some of John Wheeler's students (Everett probably among them): 'I cannot believe . . . a mouse could bring about drastic changes in the Universe simply by looking at it' (Everett 1973). Actually, as Everett replied in his PhD thesis: '. . . it is not so much the system which is effected by an observation as the observer . . . . The mouse does not effect the Universe—only the mouse is effected' (Everett 1973).

In summary, any version of the pilot wave interpretation must require the universe to move along a single trajectory. Thus all versions of the pilot wave interpretation must have ϕ quite unequal to $|\psi|^2$ at all times and this inequality must persist even in the limit of large times. But all advocates of the pilot wave interpretation admit that ϕ must approach $|\psi|^2$, at least in the limit of large time, if the pilot wave interpretation is to be consistent, and so for this reason no version of the pilot wave interpretation can be applied to quantum cosmology. I have further shown that the MWI is the only realist interpretation of quantum mechanics that allows us to interpret the wavefunction of the universe.

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